The method of coordinates II

1. Draw the points $A(4,3)$, $B(1,7)$, $C(-3,4)$, and $D(0,0)$. If you have drawn them correctly, you have the vertices of a square. What is the length of the sides of this square? What is its area? Find the coordinates of the midpoints of the sides of the square. Can you show that $ABCD$ is a square?

2. In a plane the points $A(0,0)$, $B(3,1)$, and $D(-1,2)$ are given. What coordinates must the point C have so that the quadrangle $ABCD$ will be a parallelogram?

3. Try to decide by yourself which sets of points are defined by these relations:

(a) $|x| = |y|$;

(b) $\frac{x}{|x|} = \frac{y}{|y|}$;

(c) $|x| + x = |y| + y$;

(d) $|x| + |y| = 1$;

(e) $|x| - |y| = 1$;

(f) $|x + y| + |x - y| = 2$;

(g) $x^2 - y^2 \geq 0$;

(h) $xy \geq 1$.

4. The points $A(x_1, y_1)$ and $B(x_2, y_2)$ are adjacent vertices of a parallelogram $ABCD$ with center at $O(0,0)$. What are the coordinates of points C and D?

5. In a plane the points $A(0,0)$, $B(x_1, y_1)$, and $D(x_2, y_2)$ are given. What coordinates must the point C have so that the quadrangle $ABCD$ will be a parallelogram?

6. Apply the formula for the distance between two points to prove the well-known theorem: In a parallelogram the sum of the squares of the sides is equal to the sum of the squares of the diagonals. [Hint. Problem 4 or problem 5.]

7. Using the method of coordinates, prove the following theorem: if $ABCD$ is a rectangle, then for an arbitrary point M the equality $AM^2 + CM^2 = BM^2 + DM^2$ is valid. What is the most convenient way of placing the coordinate axes?

8. What set of points is specified by the equation $x^2 + y^2 \leq 6x + 8y$?

9. Find the locus of points M the difference of the squares of whose distances from two given points A and B is equal to a given value c.

10. Find the locus of points M the sum of the squares of whose distances from the vertices of given square is equal to a given value c. For what values of c does the problem have a solution?