Algebra II

1 [122]. Factor: (a) x^4+1; (b) $a^3+b^3+c^3-3abc$.

2 [123]. Prove that if $a, b > 1$ then $a+b < 1+ab$.

3 [146]. Factor these polynomials: (a) x^4+3x^2+5x+1; (b) x^3-3x-2.

4 [152]. The polynomial P gives a remainder of $5x-7$ when divided by x^2-1. Find the remainder when P is divided by $x-1$.

5 [154]. Assume that x^3+ax^2+x+b (where a and b are some numbers) is divisible by x^2-3x+2. Find a and b.

6 [165]. Assume that $a+b+c=0$, $4a+2b+c=0$, $9a+3b+c=0$. Prove that $a=b=c=0$.

7 [224]. Solve the equation $x^2-2x-3=0$.

8 [232]. Which is bigger: $\sqrt{1001}-\sqrt{1000}$, or $1/10$?

9 [245]. Find a quadratic equation with integer coefficients having $4-\sqrt{7}$ as one of the roots.

10 [246]. The integers p, q are coefficients of the quadratic equation $x^2+px+q=0$, which has two roots. Prove that (a) the sum of squares of its roots is an integer; (b) the sum of cubes of its roots is an integer.

11 [250]. Factor $2x^2+2x+1/2$.

12 [256]. What happens with the roots of equations (a) $x^2-x-a=0$; (b) $x^2-ax+1=0$ as a changes?

13 [263]. Prove that a square has the maximum area of all rectangles having the same perimeter.

14 [264]. Prove that a square has the minimum perimeter of all rectangles having the same area.

15 [267]. Construct a biquadratic equation having exactly three solutions.